

# Massive uncharged and charged particles' tunneling from the Horowitz-Strominger Dilaton black hole

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Originally, Parikh and Wilczek's work is only suitable for the massless particles' tunneling. But their work has been further extended to the cases of massive uncharged and charged particles' tunneling recently. In this paper, as a particular black hole solution, we apply this extended method to reconsider the tunneling effect of the H.S Dilaton black hole. We investigate the behavior of both massive uncharged and charged particles, and respectively calculate the emission rate at the event horizon. Our result shows that their emission rates are also consistent with the unitary theory. Moreover, comparing with the case of massless particles' tunneling, we find that this conclusion is independent of the kind of particles. And it is probably caused by the underlying relationship between this method and the laws of black hole thermodynamics.

Key words: tunneling effect, unitary theory, laws of black hole thermodynamics.

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## I. INTRODUCTION

Since Parikh and Wilczek gave a new viewpoint on the Hawking radiation in 2000[1, 2, 3, 4], their original works have been extended to many black holes. However, all the particles that these works researched are massless, and the space-times are static[5, 6, 7, 8, 9, 10, 11, 12, 13]. Recently, Parikh and Wilczek's original method has been further extended to the following cases-the stationary space-times, massive particles' tunneling and even the charged particles' tunneling[14, 15, 16, 17, 18]. Furthermore, these works' results are also consistent with the underlying unitary theory. In this paper, as a particular black hole solution, a spherically symmetric charged dilaton black hole which is from the string theory, we reconsider the tunneling effect of the H.S Dilaton black hole[13, 21]. As we know, usually, the 3+1 dimensional charged dilaton black hole is obtained from the low energy effective field theory which describes strings, and it has many properties different from those black holes in ordinary Einstein gravity[19, 20, 22, 23] (which can be simply seen from that the calculation is more complex in Ref[13, 21]). Thus, using this extended method, we can reinvestigate the behavior of both the massive uncharged and charged particles tunneling across the event horizon of the H.S Dilaton black hole, and then calculate their corresponding emission rates. Different from the massless particles, the massive particles don't follow the radial lightlike geodesics when they tunnel across the event horizon. And even, the action for the classical forbidden trajectory should be modified for the massive charged particles, because of the existence of the electromagnetic field outside the H.S Dilaton black hole. However, if we consider the massive tunneling particle as a massive shell (de Broglie s-wave), we can investigate the behavior of both the massive uncharged and charged particles when they tunnel across the event horizon[14, 15, 18].

The rest of the paper is organized as follows. In section 2, we introduce the Painleve-H.S Dilaton coordinate system, and investigate the behavior of massive particles tunneling across the event horizon[13, 21, 24]. In section 3, first, we calculate the emission rate of massive particles, and then after modifying the action for the classical forbidden trajectory, we also calculate the emission rate of massive charged particles, and obtain their correct spectrums. In section 4, we give a conclusion and discussion on our result.

## II. PAINLEVE-H.S COORDINATES AND THE BEHAVIOR OF THE MASSIVE TUNNELING PARTICLES

The line element of the Horowitz-Strominger black hole is[13, 19, 20, 21, 22, 23]

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$$ds^2 = \left(1 - \frac{r_H}{r}\right)\left(1 - \frac{r_-}{r}\right)^{\frac{1}{1+P}} dt_h^2 - \frac{1}{\left(1 - \frac{r_H}{r}\right)\left(1 - \frac{r_-}{r}\right)^{\frac{1}{1+P}}} dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2). \quad (1)$$

where  $r_H$ ,  $r_-$  are respectively the event horizon and the inner horizon,  $P$  is a constant, and  $R(r) = r(1 - \frac{r_-}{r})^{\frac{P}{2(P+1)}}$ . In addition, the mass and the charge of the black hole are respectively

$$\begin{aligned} M &= \frac{r_H}{2} + \frac{1}{1+P} \frac{r_-}{2}, \\ Q^2 &= \frac{r_H r_- (2+P)}{2(1+P)}. \end{aligned} \quad (2)$$

In fact, if  $r_H < r_-$ , there will not even be a black hole. Thus, we set  $r_H > r_-$ . And the Painleve-H.S Dilaton black hole metric is

$$\begin{aligned} ds^2 &= (1-g)\Delta dt^2 - 2\sqrt{1-(1-g)\Delta} dt dr - dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) \\ &= g_{00}dt^2 + 2g_{01}dtdr - dr^2 - R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2). \end{aligned} \quad (3)$$

which can be obtained from the H.S Dilaton line element (1) by the coordinate transformation

$$dt = dt_h + \frac{1}{\Delta} \frac{\sqrt{1-(1-g)\Delta}}{1-g} dr. \quad (4)$$

where

$$g = \frac{r_H}{r}, \Delta = \left(1 - \frac{r_-}{r}\right)^{\frac{1}{1+P}}. \quad (5)$$

It is easy to find that the metric (3) displays the stationary, nonstatic, and nonsingular nature of the space time at the event horizon  $r_H$ . Moreover, it satisfies the Landau's condition of the coordinate clock synchronization[25, 26]. All these features are available for us to investigate the behavior of the tunneling massive particles, and which will be seen as follows.

In Parikh and Wilczek's original works, a significant key-point is to calculate the imaginary part of the action, and the first is to investigate the behavior of the tunneling particles in the corresponding Painleve coordinates[1, 2, 3, 4]. As before, the behavior of the tunneling massless particles can be given by the radial null geodesics. However, the massive particles don't follow the null geodesic when they tunnel across the event horizon, and even the massive charged particles will be the subject to Lorentz forces. In order to investigate the behavior of the tunneling massive particles, we can consider both the outgoing massive uncharged and outgoing massive charged particle as a massive shell (non-relativistic de Broglie s-wave). The difference is that the massive shell is uncharged for the massive uncharged particle, while the massive shell is charged for the massive charged particle[14, 15, 18]. On the other hand, according to the non-relativistic quantum mechanics describing the massive particles' tunneling, the de Broglie's hypothesis and the WKB formula, we can find that the behavior of the tunneling massive particles can be approximately determined by the phase velocity of the de Broglie s-wave whose phase velocity  $v_p$  and group velocity  $v_g$  have the following relationship[14, 15, 18]

$$v_p = \frac{1}{2}v_g, v_p = \frac{dr}{dt} = \dot{r}, v_g = \frac{dr_c}{dt}. \quad (6)$$

where  $r_c$  is the radial position of the particle.

Since tunneling across the barrier is an instantaneous process, there are two events that take place simultaneously in different places during the process. One event is massive particle tunneling into the barrier, and the other is massive particle tunneling out the barrier. Because the metric (3) satisfies the Landau's condition of the coordinate clock synchronization, the difference of coordinate times of these two simultaneous events is

$$dt = -\frac{g_{0i}}{g_{00}} dx^i = -\frac{g_{01}}{g_{00}} dr_c \quad (d\theta = d\varphi = 0), \quad (7)$$

Thus, according to the relationship (6), the group velocity is

$$v_g = \frac{dr_c}{dt} = -\frac{g_{00}}{g_{01}}, \quad (8)$$

and the phase velocity is

$$v_p = \dot{r} = \frac{1}{2}v_g = -\frac{1}{2}\frac{g_{00}}{g_{01}} = \frac{1}{2}\frac{(1-g)\Delta}{\sqrt{1-(1-g)\Delta}}. \quad (9)$$

Which is the behavior of both the outgoing massive uncharged and charged particles. The difference between them would be indicated if we take the self-interaction effect into consideration. Because when the particles tunnel across the event horizon of the H.S Dilaton black hole, the mass and charge in (3) and (9) should be respectively changed  $M$  with  $M - \omega$ , and  $Q$  with  $Q - q$ , where  $\omega$  and  $q$  are respectively the mass and the charge of the tunneling particle.

### III. TUNNELING RATE

#### A. Massive uncharged particles' tunneling rate

According to Parikh and Wilczek's original works, the tunneling rate  $\Gamma$  could take the following form[2, 27]

$$\Gamma \sim \exp(-2\text{Im}S). \quad (10)$$

and after using the Hamilton's equation  $\frac{dH}{dp_r} = \dot{r}$ , the imaginary part of the action is

$$\text{Im}S = \text{Im} \int_{r_i}^{r_f} p_r dr = \text{Im} \int_{r_i}^{r_f} \int_0^{p_r} dp_r' dr = \text{Im} \int_{r_i}^{r_f} \int_{M_i}^{M_f} \frac{dM}{\dot{r}} dr. \quad (11)$$

Where  $M_i$  and  $M_f$  are respectively the initial mass and the final mass of the black hole.

Thus, keeping  $Q$  a constant, substituting (9) into (11), and then doing the  $r$  integral first, we obtain

$$\text{Im}S = \text{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{2\sqrt{r^2 - (r - r_H)r\Delta} dr dM}{(r - r_H)(1 - \frac{r_-}{r})^{\frac{1}{1+P}}} = -\pi \int_{M_i}^{M_f} \frac{2r_H}{(1 - \frac{r_-}{r_H})^{\frac{1}{1+P}}} dM. \quad (12)$$

Not as before, we don't integral (12) directly. Instead, we first calculate the differential of the entropy  $S$  to the mass  $M$ . And the entropy of the H.S Dilaton black hole is[28]

$$\begin{aligned} S &= \frac{1}{4}A = \pi R^2(r_H) = \pi r_H^2 (1 - \frac{r_-}{r_H})^{\frac{P}{1+P}} \\ &= \pi [r_H^2 - Q^2 \frac{2(1+P)}{2+P}]^{\frac{P}{1+P}} r_H^{\frac{2}{1+P}}. \end{aligned} \quad (13)$$

Therefore, the differential of the entropy  $S$  to the mass  $M$  is

$$dS = \frac{dS}{dr_H} \frac{dr_H}{dM} dM = \frac{4\pi r_H}{(1 - \frac{r_-}{r_H})^{\frac{1}{1+P}}} dM. \quad (14)$$

From (14) and (12), we can easily obtain

$$\Gamma \sim e^{-2\text{Im}S} = e^{\Delta S}. \quad (15)$$

which is consistent with the underlying unitary theory and support the conservation of information.

### B. Massive charged particles' tunneling rate

When the massive charged particles tunnel across the event horizon of the H.S Dilaton black hole, not only the mass but also the charge of black hole will change for the conservation of energy and charge. In addition, different from the massive uncharged particles, for the existence of the electromagnetic field outside the H.S Dilaton black hole, we should also take its effect into account when the massive charged particles tunnel across the event horizon. Therefore, we must consider the black hole and the electromagnetic field outside black hole as a whole matter-gravity system[14, 15, 18]. For the H.S Dilaton black hole, the 4-dimensional electromagnetic potential is

$$A_\mu = (A_t, 0, 0, 0). \quad (16)$$

where  $A_t = -Q/r$ . And the Lagrangian function of the electromagnetic field corresponding to the generalized coordinates described by  $A_\mu$  is  $L_e = -1/4 F_{\mu\nu} F^{\mu\nu}$ . Thus, when a massive charged particle tunnels out, the whole system will transit from one state to another. However, we find that the generalized coordinate  $A_t$  is an ignorable coordinate. In order to obtain the correct kinematical equation from the differential of the action, we should eliminate the freedom corresponding to  $A_t$ . Therefore, the action of the charged massive particle should be written as

$$\begin{aligned} S &= \int_{t_i}^{t_f} (P_r \dot{r} - P_{A_t} \dot{A}_t) dt \\ &= \int_{r_i}^{r_f} \left[ \int_{(0,0)}^{(P_r, P_{A_t})} (\dot{r} dP'_r - \dot{A}_t dP'_{A_t}) \right] \frac{dr}{r}. \end{aligned} \quad (17)$$

According to the Hamilton's equations, we have

$$\begin{aligned} \dot{r} &= \frac{dH}{dP_r} \Big|_{(r; A_t, P_{A_t})} = \frac{dM}{dP_r}, \\ \dot{A}_t &= \frac{dH}{dP_{A_t}} \Big|_{(A_t; r, P_r)} = \frac{Q}{r} dQ. \end{aligned} \quad (18)$$

Substituting (18) into (17), and switching the order of integration yield the imaginary part of the action

$$\text{Im}S = \text{Im} \int_{r_i}^{r_f} \int_{(M, Q)}^{(M-\omega, Q-q)} \left[ dM - \frac{Q}{r} dQ \right] \frac{dr}{r}. \quad (19)$$

We would like to emphasize that, since we consider the massive charged particles' tunneling, the  $Q$  in the expression of  $\dot{r}$  in (9) should also be changed. In addition, we also can find that the conservation of energy and electric charge will be enforced in a natural way. Thus, substituting the (9) into (19), and doing the  $r$  integral first, we obtain

$$\begin{aligned} \text{Im}S &= \text{Im} \int_{r_i}^{r_f} \int_{(M, Q)}^{(M-\omega, Q-q)} \left[ dM - \frac{Q}{r} dQ \right] \frac{2\sqrt{r^2 - (r - r_H)r\Delta} dr}{(r - r_H)(1 - \frac{r_-}{r})^{\frac{1}{1+P}}} \\ &= -\pi \int_{(M, Q)}^{(M-\omega, Q-q)} \left[ \frac{2r_H}{(1 - \frac{r_-}{r_H})^{\frac{1}{1+P}}} dM - \frac{2Q}{(1 - \frac{r_-}{r_H})^{\frac{1}{1+P}}} dQ \right]. \end{aligned} \quad (20)$$

Similarly, we don't directly calculate the integral (20). We first calculate the differential of the entropy  $S$  to both the mass  $M$  and the electric charge  $Q$ . According to the expression of the entropy in (13), the complete differential of the entropy is

$$\begin{aligned} dS &= \frac{\partial S}{\partial r_H} \frac{\partial r_H}{\partial M} dM + \left( \frac{\partial S}{\partial r_H} \frac{\partial r_H}{\partial Q} + \frac{\partial S}{\partial Q} \right) dQ \\ &= \frac{4\pi r_H}{(1 - \frac{r_-}{r_H})^{\frac{1}{1+P}}} dM - \frac{4\pi Q}{(1 - \frac{r_-}{r_H})^{\frac{1}{1+P}}} dQ. \end{aligned} \quad (21)$$

Comparing (21) with (20), we can also easily obtain the equation (15) and the same conclusion, and find that the result takes the same functional form as that of the massive uncharged particles.

#### IV. CONCLUSION AND DISCUSSION

Recently, the charged dilaton black holes have been much researched. As is mentioned in section one, the charged dilaton black holes have many properties different from those black holes in ordinary Einstein gravity. Thus, in this paper, using the extended method[14, 15, 18], we first investigate the behavior of both the massive uncharged and charged particles which tunnel across the event horizon of the H.S charged dilaton black hole, and then reconsider the tunneling effect of it. But our result shows that their emission rates are also consistent with the unitary theory, the same as that of the case of massless particles' tunneling, which manifests that the conclusion is independent of the kind of particles. On the other hand, as is seen from the Refs[13, 21], for the massless particles' tunneling of the H.S Dilaton black hole, the authors do directly calculate the imaginary part of the action after doing the  $r$  integral, and this direct calculation is usually complex. However, in our paper, we simplify the calculation. That, after doing the  $r$  integral, we don't directly calculate it. Instead, we calculate the differential of the entropy  $S$  to the variance, compare it with the imaginary part of the action after doing the  $r$  integral, and then we can easily obtain the same equation (15) and conclusion. But why is the conclusion independent of the kind of particles? does this two sides have relationship? In the following part, we will give a discussion on these two questions. Our discussion shows that, in viewing of the laws of black hole thermodynamics, these two questions may be solved well, which is explained in detail in the following.

As we know, according to the first law of black hole thermodynamics, the general differential Bekenstein-Smarr equation of the black hole is[29]

$$dM = \frac{\kappa}{8\pi}dA + VdQ + \Omega dJ \quad (22)$$

Therefore, if we consider that the tunnelling process is a reversible process, according to the second law of black hole thermodynamics, (22) can be rewritten as

$$dM = TdS + VdQ + \Omega dJ \quad (23)$$

which is equivalent to

$$dS = \frac{dM}{T} - \frac{VdQ}{T} - \frac{\Omega dJ}{T} \quad (24)$$

In our cases, we investigate the behavior of the particles tunneling across the event horizon of the H.S Dilaton black hole. The temperature  $T$  and the electro-potential  $V$  are[20]

$$T = \frac{1}{4\pi} \left(1 - \frac{r_-}{r_+}\right)^{\frac{1}{1+p}}, V = \frac{Q}{r}. \quad (25)$$

Thus, when we consider the massive uncharged or charged particles tunneling across the event horizon without the angle momentum, the equations corresponding to (24) are just respectively the (14) and the (21). Moreover, in Refs[13, 21], we can also obtain the corresponding equation from (24). It implies that not only our simplified calculation has hidden the laws of black hole thermodynamics, but also all those methods (either the original Parikh and Wilczek's method or its extended method) have the underlying relationship with the laws of black hole thermodynamics. As we know, the laws of black hole thermodynamics are independent of the kind of tunneling particles, thus, the independence of the conclusion may come from it. (Note that, we had already found the conclusion's independence of the kind of particles in the Ref[30]. And concerning this independence, we gave a brief discussion. However, this discussion is not complete. Thus, this paper can also be considered as the advanced discussion about the independence).

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